Geometric Measure Theory I+II

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Background and purpose A very successful strategy for the study of geometric variational problems is to firstly prove existence in an enlarged class of competitors by means of compactness theorems and subsequently study the regularity of the solution therein. For instance, instead of considering only smooth submanifolds, one proves existence in the classes of boundaries of sets of finite perimeter, integral currents, or integral varifolds—all of which are based on the more basic concept of rectifiable set. In the ensuing regularity theory, the generality of varifolds allows to unify a substantial part of the treatment. The purpose of the course is to develop, after providing the necessary infrastructure, the concept of rectifiable set as well as key elements of the theory of varifolds.

Prerequisites We assume sound familiarity with the concepts of measure, measurable function, Lebesgue integration, and product measure. *Students wishing to review this material before the course are encouraged to contact the teacher by email for relevant lecture notes on Real Analysis, see [Men19].*

Course outline In the initial part of the course, we focus on developing the relevant concepts from advanced measure theory (Riesz representation theorem, covering theorems, derivatives of measures), functional analysis (locally convex spaces, weak topology), multilinear algebra (exterior algebra, alternating forms) and basic submanifold geometry (second fundamental form, Grassmann manifold). Then, we develop Hausdorff measures and the area of Lipschitzian maps to treat rectifiable sets. Finally, the basic theory of varifolds is developed (first variation, monotonicity identity) to treat the isoperimetric inequality and compactness theorems.

Sources for the course and other information on the course Our main sources are [Fed69] and, concerning varifolds, [All72] supplemented by more detailed LaTeX-ed lecture notes; some simplifications from [Men16] and [MS18] will also be employed. A survey of the concept of varifold is available at [Men17].

(1) The course is offered both at the National Taiwan Normal University and in the programme of the Taiwan Mathematics School.

(2) Video-recordings of the lectures will be made available to the participants of the course.

References

[All72] William K. Allard. On the first variation of a varifold. *Ann. of Math. (2)*, 95:417–491, 1972. URL: `https://doi.org/10.2307/1970868`.

[Fed69] Herbert Federer. *Geometric measure theory.* Die Grundlehren der mathematischen Wissenschaften, Band 153. Springer-Verlag New York Inc., New York, 1969. URL: `https://doi.org/10.1007/978-3-642-62010-2`.

[Men16] Ulrich Menne. Weakly differentiable functions on varifolds. *Indiana Univ. Math. J.*, 65(3):977–1088, 2016. URL: `https://doi.org/10.1512/iumj.2016.65.5829`.

[Men17] Ulrich Menne. The concept of varifold. *Notices Amer. Math. Soc.*, 64(10):1148–1152, 2017. URL: `https://doi.org/10.1090/noti1589`.

[Men19] Ulrich Menne. Real Analysis, 2019. Lecture notes, National Taiwan Normal University.

[MS18] Ulrich Menne and Christian Scharrer. An isoperimetric inequality for diffused surfaces. *Kodai Math. J.*, 41(1):70–85, 2018. URL: `https://doi.org/10.2996/kmj/1521424824`.

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