Topics in Geometric Analysis I+II

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Background and purpose In *Geometric Analysis* geometric problems are studied by the methods of analysis. Whereas many of these problems can be formulated in the language of smooth functions and submanifolds, their solution often gives rise to less restrictive classes of objects some of which shall be introduced in this course.

Prerequisites We assume familiarity with the concepts of abstract measure and Lebesgue integration.

Course outline Topics are presented by the participants and are assigned in consultation with the teachers taking into account the prior knowledge of the individual participants. Amongst the possible topics are the following.

(1) Computations and geometric measure theory, see for instance [BLM17] and [CT13]; this leaves plenty of choices.

(2) Functions of bounded variation, see [AFP00] or [Mag12]; this also leaves plenty of choices.

(3) Topological vector spaces, see [Bou87], with emphasis on the items summarised in [Men16, Chapter 2].

(4) An example concerning approximate differentiation, see [Koh77].

(5) Pointwise differentiability of higher order for sets, see [Men19].

(6) Rectifiability and approximate differentiability of higher order for sets, see [San19].

(7) An example of a Borel set in $\mathbf{R}^2 \simeq \mathbf{R} \times \mathbf{R}$ whose orthogonal projection onto $\mathbf{R}$ is not a Borel set, see [Fed69, 2.2.9, 2.2.11].

Items (5), (6), and (7) may be particularly suitable for students who followed the course *Special Topics in Analysis* in the preceding term.

References

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